General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MPC3

Unit Pure Core 3

Thursday 31 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 Use the mid-ordinate rule with four strips to find an estimate for $\int_{0.4}^{1.2} \cot(x^2)\,dx$, giving your answer to three decimal places. *(4 marks)*

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2 For $0 < x \leqslant 2$, the curves with equations $y = 4\ln x$ and $y = \sqrt{x}$ intersect at a single point where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.5. *(2 marks)*

(b) Show that the equation $4\ln x = \sqrt{x}$ can be rearranged into the form

$$x = e^{\left(\frac{\sqrt{x}}{4}\right)}$$ *(1 mark)*

(c) Use the iterative formula

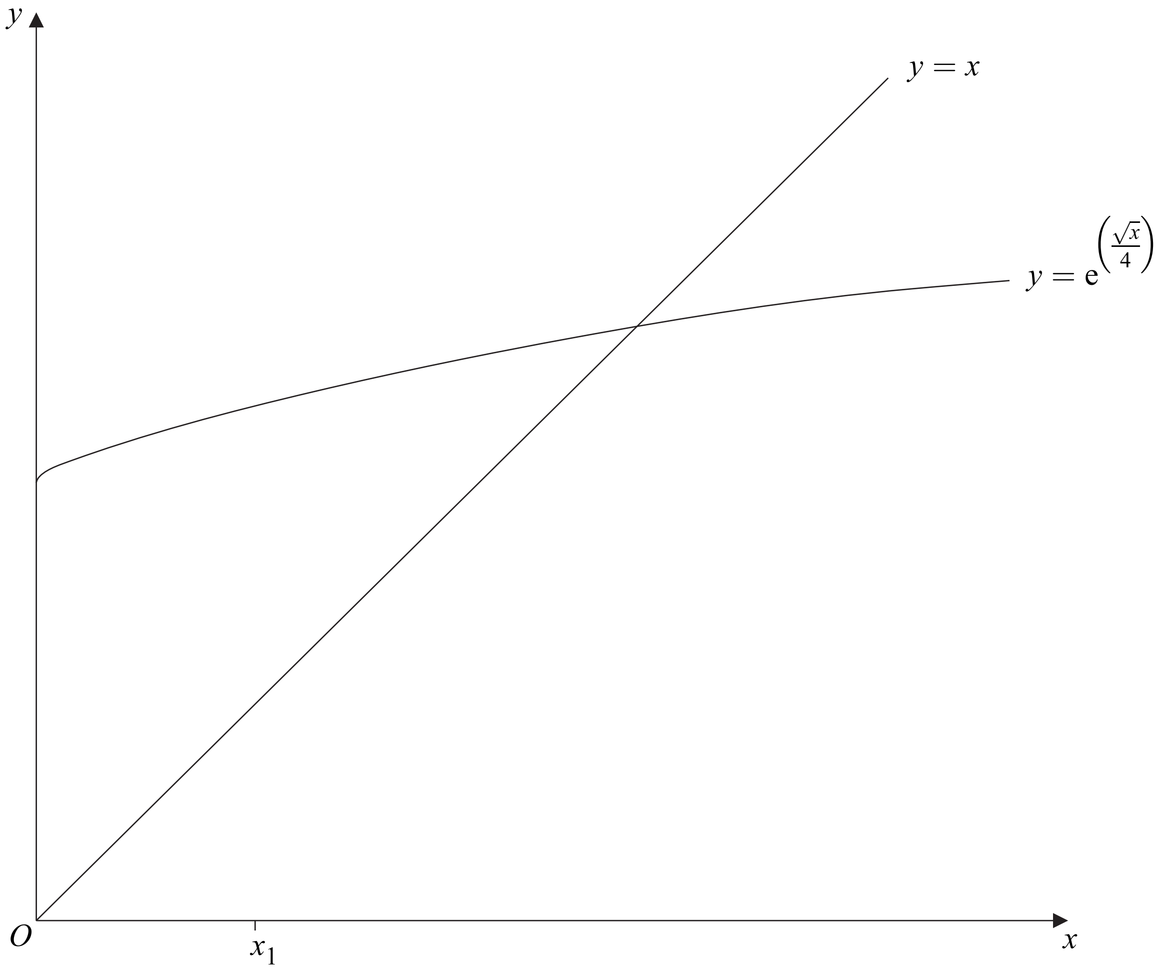
$$x_{n+1} = e^{\left(\frac{\sqrt{x_n}}{4}\right)}$$

with $x_1 = 0.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(d) **Figure 1**, on the page 3, shows a sketch of parts of the graphs of $y = e^{\left(\frac{\sqrt{x}}{4}\right)}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

Figure 1



3 A curve has equation $y = x^3 \ln x$.

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) (i) Find an equation of the tangent to the curve $y = x^3 \ln x$ at the point on the curve where $x = e$. *(3 marks)*

(ii) This tangent intersects the x-axis at the point A. Find the exact value of the x-coordinate of the point A. *(2 marks)*

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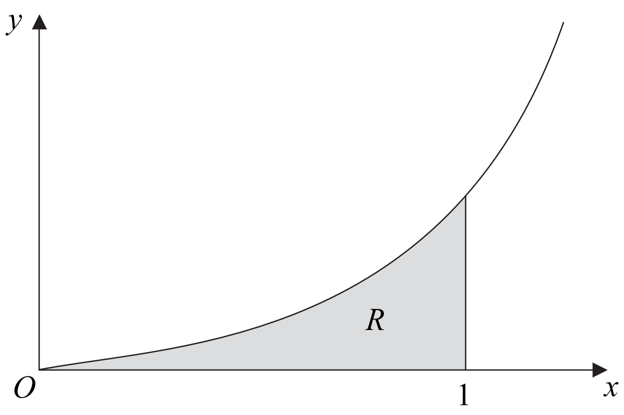
4 (a) By using integration by parts, find $\int x\,e^{6x}\,dx$. *(4 marks)*

(b) The diagram shows part of the curve with equation $y = \sqrt{x}\,e^{3x}$.



The shaded region R is bounded by the curve $y = \sqrt{x}\,e^{3x}$, the line $x = 1$ and the x-axis from $x = 0$ to $x = 1$.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in the form $\pi(pe^6 + q)$, where p and q are rational numbers. *(3 marks)*

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5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{2x - 5}, \quad \text{for } x \geqslant 2.5$$

$$g(x) = \frac{10}{x}, \quad \text{for real values of } x, \quad x \neq 0$$

(a) State the range of f. *(2 marks)*

(b) (i) Find $fg(x)$. *(1 mark)*

(ii) Solve the equation $fg(x) = 5$. *(2 marks)*

(c) The inverse of f is f^{-1}.

(i) Find $f^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $f^{-1}(x) = 7$. *(2 marks)*

6 Use the substitution $u = x^4 + 2$ to find the value of $\int_0^1 \frac{x^7}{(x^4+2)^2}\,\mathrm{d}x$, giving your answer in the form $p \ln q + r$, where p, q and r are rational numbers. *(6 marks)*

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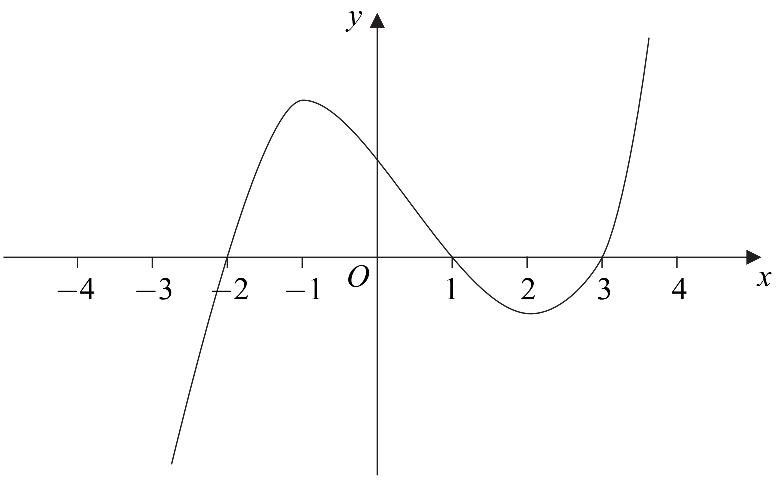
7 The sketch shows part of the curve with equation $y = \mathrm{f}(x)$.



(a) On **Figure 2** on page 6, sketch the curve with equation $y = |\mathrm{f}(x)|$. *(3 marks)*

(b) On **Figure 3** on page 6, sketch the curve with equation $y = \mathrm{f}(|x|)$. *(2 marks)*

(c) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \frac{1}{2}\mathrm{f}(x+1)$. *(4 marks)*

(d) The maximum point of the curve with equation $y = \mathrm{f}(x)$ has coordinates $(-1, 10)$. Find the coordinates of the maximum point of the curve with equation $y = \frac{1}{2}\mathrm{f}(x+1)$. *(2 marks)*

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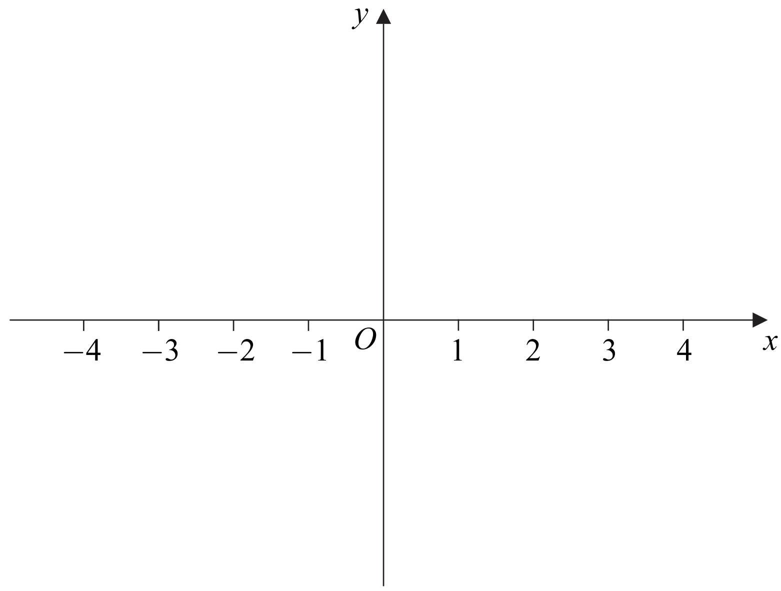
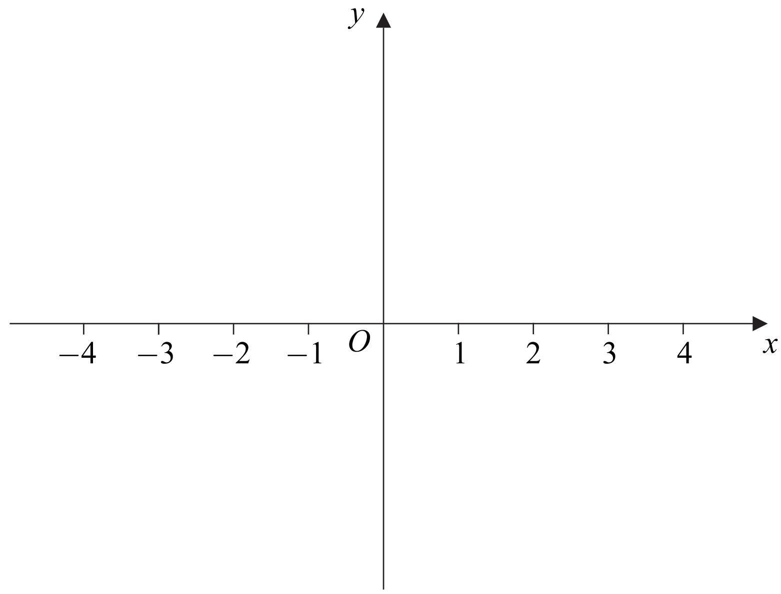
(a)

Figure 2



(b)

Figure 3



8 (a) Show that the equation

$$\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}=32$$

can be written in the form

$$\operatorname{cosec}^2\theta = 16$$ *(4 marks)*

(b) Hence, or otherwise, solve the equation

$$\frac{1}{1+\cos(2x-0.6)}+\frac{1}{1-\cos(2x-0.6)}=32$$

giving all values of x in radians to two decimal places in the interval $0 < x < \pi$.
(5 marks)

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9 (a) Given that $x = \dfrac{\sin y}{\cos y}$, use the quotient rule to show that

$$\frac{dx}{dy} = \sec^2 y$$ *(3 marks)*

(b) Given that $\tan y = x - 1$, use a trigonometrical identity to show that

$$\sec^2 y = x^2 - 2x + 2$$ *(2 marks)*

(c) Show that, if $y = \tan^{-1}(x-1)$, then

$$\frac{dy}{dx} = \frac{1}{x^2 - 2x + 2}$$ *(1 mark)*

(d) A curve has equation $y = \tan^{-1}(x-1) - \ln x$.

(i) Find the value of the x-coordinate of each of the stationary points of the curve.
(4 marks)

(ii) Find $\dfrac{d^2y}{dx^2}$. *(2 marks)*

(iii) Hence show that the curve has a minimum point which lies on the x-axis. *(2 marks)*